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NGHIÊN CỨU MÔ HÌNH HÓA ĐỘNG HỌC NGƯỢC CÁNH TAY ROBOT DƯ DẪN ĐỘNG DỰA TRÊN KỸ THUẬT ĐIỀU KHIỂN VÒNG KÍN

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TỪ KHÓA

Cánh tay robot;
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Phản hồi vòng kín.

TÓM TẮT

Bài báo này trình bày kết quả phân tích vấn đề động học ngược (IK) của cánh tay robot PA10-7C có bảy độ tự do (7DOF) dựa trên kỹ thuật phản hồi vòng kín (CLK). Lý thuyết DH được sử dụng để xây dựng mô hình toán học của robot. Phương trình động học ngược được xây dựng để biểu diễn mối quan hệ giữa vị trí và hướng của điểm kết thúc (EE) trong không gian làm việc với các tọa độ tổng quát của các khớp trong không gian khớp. Thuật toán CLF được trình bày và sử dụng để phân tích vấn đề IK củ a robot với đường đi đầu vào là vị trí của điểm EE trong không gian làm việc. Kết quả phân tích bài toán IK cho thấy sai số vị trí lớn nhất của điểm thao tác cuối đạt $3.7x10^{-4}$ mm và giá trị này nhỏ hơn rất nhiều so với chiều dài quỹ đạo di chuyển (400mm). Điều này chứng tỏ hiệu quả cao của thuật toán CLK đối với hệ thống dư dẫn động. Thuật toán này có thể được sử dụng làm cơ sở cho các vấn đề động học và điều khiển cho loại hệ thống này.

INVERSE KINEMATIC ANALYSIS OF A REDUNDANT ROBOT ARM BASED ON CLOSED-LOOP FEEDBACK ALGORITHM

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ABSTRACT

This article presents inverse kinematic (IK) problem analysis results of the PA10-7C robot arm with seven degrees of freedom (7DOF) based on closed-loop feedback (CLK) technique. The DH theory is used to establish the mathematical model of the robot. Kinematic equations are built to express the relationship between the position and direction of the end-effector (EE) point in the workspace with the generalized coordinates of the joints in the joint space. The CLF algorithm is presented and used to analyze the IK problem of the robot with the input trajectory being the position of the EE point in the workspace. The analysis results of the IK problem show that the maximum position error of the last operating point reaches $3.7x10^{-4}$ mm and this value is much smaller than the length of the moving trajectory (400mm). This demonstrates the high effectiveness of the CLK algorithm for redundant systems. This algorithm can be used as a basis for dynamics and control problems for this type of system.

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1. INTRODUCTION

The characteristics of the redundant robot create the distinctive of the inverse kinematics problem. It means that will exist countless options for the configuration of the robot in response to a task in the workspace, or in other words, the system of inverse kinematics (IK) equations has countless solutions. This is the advantage of this type of robot because we can choose an optimal result among a multitude of results on the basis of applying quality criteria such as avoidance of limited joints, joint velocity, avoid obstacles, the configuration degradation and is quite useful for minimizing energy consumption [1]. However, due to the multitude of solutions to choose from, choosing a viable option that fits the robot's physical configuration is a huge challenge. Research on effective solutions to solve the IK problem for redundant robots is the top concern because it is the foundation to analyze the dynamics problem and design the control system. There are two basic solutions to solve the IK problem. It is the analytic method and the numerical method. The analytical method has the advantage of providing a tremendous accurate solution, but it is really difficult to apply to solve redundant robot systems because of the complexity of the system. The numerical method is increasingly being developed with the advancement of computer science. Basically, the numerical approach can be applied to solve any kind of robot configuration without too much difficulty. The disadvantage of the method is that the solution is just approximate. However, with today's mathematical tools and scientific developments, the error of the solution is extremely small, completely meeting the robot motion tasks in practice.

There are many algorithms which are developed to solve the inverse kinematics problems such as Jacobian Transpose [2], Pseudoinverse [3], Damped Least Squares [4], Quasi-Newton and conjugate gradient [5], Closed-loop inverse kinematics (CLIK) [6]. The inverse kinematics problem was solved by using CLIK method with the velocity and acceleration constraints [7], [8]. CLIK method is used in [9] for welding robot 6-DOFs combining with a positioner to track a complex 3D curve. The parallel genetic algorithm is used to solve IK problem of Puma 500 robot in [10]. The elimination technique is presented in [11] to reduce the complexity of inverse kinematic formulation based on analytical method. The new solution method is introduced in [12] to avoid joint limitation, avoid singularities and avoid obstacles for redundant robots.

This article focuses on describing the process of mathematical modeling of the robot arm PA10-7C with 7DOF and the results of IK problem based on CLK technique. The structure of the article includes specific sections such as Section1: Introduction; Section 2: Kinematics modelling; Section 3: CLK algorithm; Section 4: Numerical simulation results and IK analysis.

2. KINEMATICS MODELLING OF REDUNDANT ROBOT ARM

Consider the kinematics model of PA10-7C robot with 7-DOF as shown in Fig. 1. The fixed coordinate system is $(OXYZ)_0$ and $(OXYZ)_i$ ($i = 1 \div 7$) are the local coordinate systems attached link i .

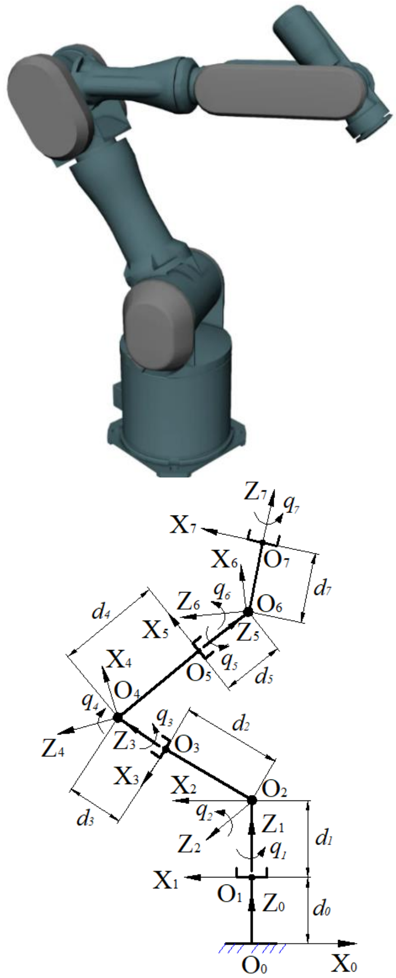

Figure 1. *Kinematics model PA10-7C*

Tab. 1 describes the kinematic parameters according to the D-H rule [13] of PA10-7C robot. Accordingly, the transformation $\mathbf{H}_i$ homogeneous matrices are determined.

Table 1. *D-H parameters PA10-7C*

Links	D-H parameters			
	θ_i	d_i	a_i	α_i
1	q_1	$d_0 + d_1$	0	$-\pi/2$
2	q_2	d_2	0	$\pi/2$
3	q_3	d_3	0	$-\pi/2$
4	q_4	d_4	0	$\pi/2$
5	q_5	d_5	0	$-\pi/2$
6	q_6	0	0	$\pi/2$
7	q_7	d_7	0	0

Where, $\mathbf{q} = [q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5 \quad q_6 \quad q_7]^T$ is the generalized vector of joints and geometry parameters of robot arm such as $d_0 = 0.21(m)$, $d_1 = 0.1(m)$, $d_2 = 0.21(m)$, $d_3 = 0.13(m)$, $d_4 = 0.28(m)$, $d_5 = 0.21(m)$, $d_7 = 0.07(m)$. The velocity vector of joints is $\dot{\mathbf{q}} = \begin{bmatrix} \dot{q}_1 & \dot{q}_2 & \ldots & \dot{q}_7 \end{bmatrix}^T (rad/s)$. Based on the D-H method, a local D-H matrix $\mathbf{H}_i$ for the link i was established as follows

$$\mathbf{H}_i = \begin{bmatrix} \mathbf{A}_i & \mathbf{x}_i \\ 0 & 1 \end{bmatrix} \quad (1)$$

This matrix shows the position ($\mathbf{x}_i$) and the cosine matrix indicates the direction $\mathbf{A}_i$ of the link i relative to the local coordinate system $(OXYZ)_{i-1}$. For a robot consisting of n serial of links (in this case, $n = 7$), the position of the end-effector point on link n relative to the fixed coordinate system is determined as

$$\mathbf{D}_n = \mathbf{H}_1 \mathbf{H}_2 .. \mathbf{H}_n \quad (2)$$

The $\mathbf{D}_n$ matrix is obtained as

$$\mathbf{D}_n = \begin{bmatrix} \mathbf{A}_n & \mathbf{x}_n \\ 0 & 1 \end{bmatrix} \quad (3)$$

Where, $\mathbf{A}_n$ the cosine matrix indicates the direction, $\mathbf{x}_n = \mathbf{x} = \begin{bmatrix} x_E & y_E & z_E \end{bmatrix}^T$ is the vector which describes position of the end-effector point relative to the fixed coordinate system. In essence, the vector $\mathbf{x}$ represents the relationship between the end-effector point position and the joint variable values. Therefore, the forward kinematics equations are determined as $\mathbf{x} = f(\mathbf{q})$ and the end-effecter point velocity is inferred as $\dot{\mathbf{x}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}$. The Jacobian matrix is $\mathbf{J}(\mathbf{q})$ with size $3 \times n$. The inverse kinematics equations are determined as follows

$$\mathbf{q}(t) = f^{-1}(\mathbf{x}(t)) \quad (4)$$

Once the values of $\mathbf{q}$ have been determined from equation (4), the joints velocity are calculated as

$$\dot{\mathbf{q}} = \mathbf{J}^{+}(\mathbf{q})\dot{\mathbf{x}} \quad (5)$$

Where, $\mathbf{J}^{+}(\mathbf{q})$ is the pseudo-inverse matrix of $\mathbf{J}(\mathbf{q})$ and defined as [13]

$$\mathbf{J}^{+}(\mathbf{q}) = \mathbf{J}^T(\mathbf{q})(\mathbf{J}(\mathbf{q})\mathbf{J}^T(\mathbf{q}))^{-1} \quad (6)$$

The joints acceleration is inferred from (6) as

$$\ddot{\mathbf{q}} = \mathbf{J}^{+}(\mathbf{q})(\ddot{\mathbf{x}} - \dot{\mathbf{J}}\dot{\mathbf{q}}) \quad (7)$$

After calculating the value of vector $\mathbf{q}$, the value of vectors $\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$ can be determined according to the Eq. 5 and Eq. 7.

3. CLF ALGORITHM

Assume the given the EE point trajectory and its velocity are $\mathbf{x}_d(t) = \begin{bmatrix} x_E & y_E & z_E \end{bmatrix}^T ; \dot{\mathbf{x}}_d(t) = \begin{bmatrix} \dot{x}_E & \dot{y}_E & \dot{z}_E \end{bmatrix}^T$. The goal is to find possible joint variable values in the joint space $\mathbf{q}(t), \dot{\mathbf{q}}(t)$ to ensure the trajectory requirements set with the joint variable vector is $\mathbf{q} = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 & q_5 & q_6 & q_7 \end{bmatrix}^T$. It is easy to see that the number of joint variables to be found is larger than the given number of position variables in the workspace. Therefore, this is a redundant system. The forward kinematics equations are determined as follows

$$\dot{\mathbf{x}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} \quad (8)$$

The kinematics differential equations can be written as follows

$$\dot{\mathbf{x}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} \quad (9)$$

Jacobian matrix $\mathbf{J}(\mathbf{q})$ has size $3 \times n$ and is not a square matrix

$$\mathbf{J}(\mathbf{q}) = \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} & .... & J_{1n} \\ J_{21} & J_{22} & J_{23} & J_{24} & .... & J_{2n} \\ J_{31} & J_{32} & J_{33} & J_{34} & .... & J_{3n} \end{bmatrix} \quad (10)$$

The main solution to solve the IK problem from Eq. 7 is to use a quasi-inverse matrix $\mathbf{J}^*(\mathbf{q})$ from matrix $\mathbf{J}(\mathbf{q})$ [14].

$$\dot{\mathbf{q}} = \mathbf{J}^*(\mathbf{q})\dot{\mathbf{x}} \quad (11)$$

In which, the quasi-inverse matrix is defined as follows

$$\mathbf{J}^* = \mathbf{J}^T(\mathbf{J}\mathbf{J}^T)^{-1} \quad (12)$$

With redundatn robot systems, the commonly used method is presented in [14] as

$$\dot{\mathbf{q}} = \mathbf{J}^*(\mathbf{q})\dot{\mathbf{x}} + (\mathbf{I} - \mathbf{J}^*(\mathbf{q})\mathbf{J}(\mathbf{q}))\dot{\mathbf{q}}_0 \quad (13)$$

In which, $\mathbf{I}$ is the unit matrix of size $n \times n$. However, reality shows that this algorithm sometimes still does not give accurate results because there is no feedback signal. Consider a CLK algorithm that allows bias feedback on the position and velocity of the EE point in the workspace as follows

$$\mathbf{e} = \mathbf{x}_d - \mathbf{x}; \dot{\mathbf{e}} = \dot{\mathbf{x}}_d - \dot{\mathbf{x}} \quad (14)$$

Solving the IK problem is converted to finding the variables value in the joint space that matches the working requirements in the workspace and bringing the position and velocity errors to zero value. Solution the redundant problem is expressed as follows

$$\dot{\mathbf{q}} = \mathbf{J}^{*}(\mathbf{q})(\dot{\mathbf{x}}_d + \mathbf{K}_p(\mathbf{x}_d - \mathbf{x})) \tag{15}$$

The error equation is obtained from combining Eq. 14 and Eq. 15 as follows

$$\dot{\mathbf{e}} + \mathbf{K}_p \mathbf{e} = 0 \tag{16}$$

In which, $\mathbf{K}_p$ is a diagonal matrix, constant and positive definite. Determining the value of this matrix will determine how quickly or slowly the error reaches 0. Tracking position and velocity values in joint space and workspace is an advantage that open-loop algorithms cannot do. The final defined algorithm is as follows

$$\dot{\mathbf{q}} = \mathbf{J}^{*}(\mathbf{q})(\dot{\mathbf{x}}_d + \mathbf{K}_p(\mathbf{x}_d - \mathbf{x})) + (\mathbf{I} - \mathbf{J}^{*}(\mathbf{q})\mathbf{J}(\mathbf{q}))\dot{\mathbf{q}}_0 \tag{17}$$

Implement the algorithm in the MATLAB/SIMULINK environment as shown in Fig. 2.

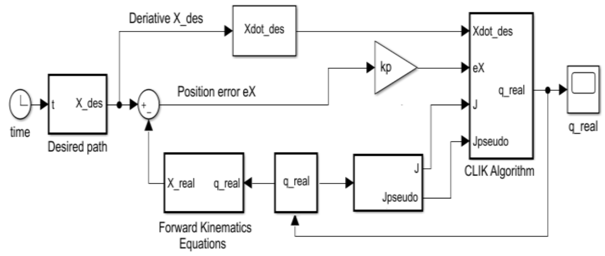

Figure 2. *CLF algorithm model implemented in MATLAB/SIMULINK*

4. THE IK PROBLEM ANALYZING RESULTS

A given trajectory in the workspace for PA10-7C robot is presented as (Fig. 3)

$$\begin{aligned} x_E &= 0.2 + 0.4\sin(0.2t)(m); \\ y_E &= 0.2 + 0.4\cos(0.2t)(m); \\ z_E &= 0.65(m) \end{aligned} \tag{18}$$

Accordingly, the EE point of the robot moves in workspace with constant coordinates along the OZ axis. This makes verifying the results more convenient than other more complex trajectories.

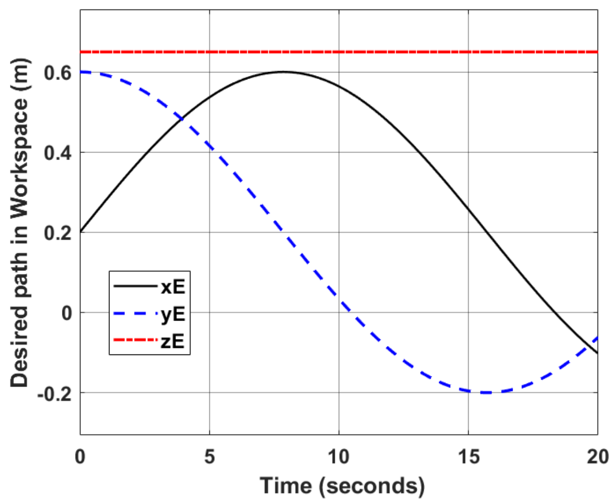

Figure 3. *The EE point trajectory in the workspace*

Correspondingly, the velocity of the EE point in the workspace along the coordinate axes is described as follows (Fig. 4)

$$\begin{aligned} dx_E &= \dot{x}_E = 0.08\cos(0.2t)(m); \\ dy_E &= \dot{y}_E = -0.08\sin(0.2t)(m); \\ dz_E &= \dot{z}_E = 0 \end{aligned} \tag{19}$$

The algorithm diagram in Fig. 2 is implemented on MATLAB/SIMULINK software with coefficient kp=50, sampling time is 0.01s.

The inverse kinematic analysis results of the robot arm PA10-7C with 7DOF are shown through the kinematics behavior of the joints including joint position, velocity and acceleration. These results are depicted in Fig. 5, Fig. 6, Fig. 7 and Fig. 8.

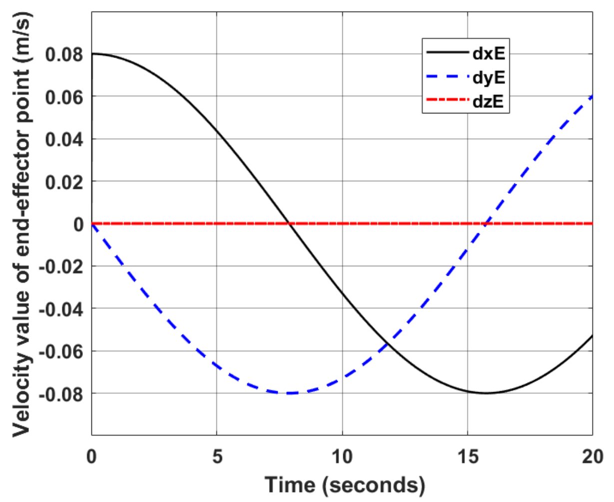

Figure 4. *EE point velocity value in the workspace*

Fig. 5 depicts the position of joints corresponding to the motion trajectory of the required EE point. It is easy to see that the values of the joints are all within the allowable limits. Accordingly, joint 1 moves with the largest joint angle reaching 4.1 (rad). In contrast, joint 7 does not participate in movement because this joint is attached to the last link and the EE point.

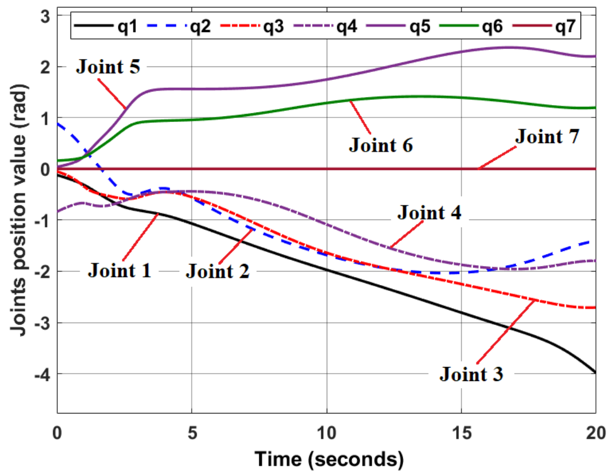

Figure 5. *Joint position values*

However, Fig. 6 depicts the velocity of the joints. Accordingly, joint 4 has the highest velocity value of 0.72 (rad/s), followed by joint 2 with 0.65 (rad/s). Joint 6 and joint 3 have similar maximum velocity values (0.42 (rad/s) and 0.43 (rad/s)). The velocity value tends to decrease as the EE point gradually approaches the end of the trajectory.

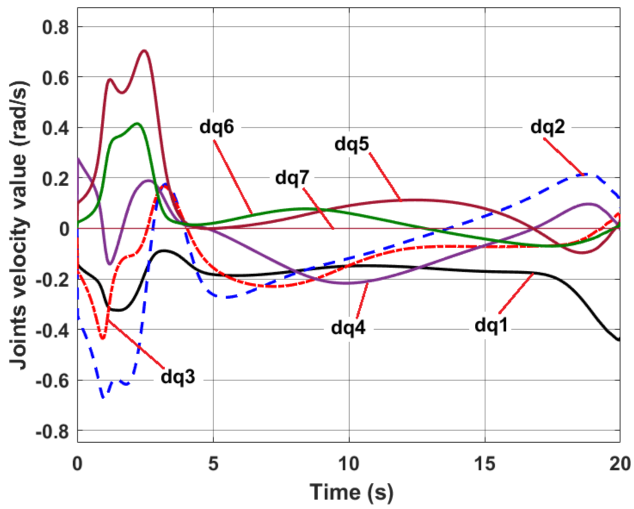

Figure 6. *Joint velocity values*

Fig. 7 shows the acceleration of the joints. The acceleration value of joint 5 is the largest (1.2 rad/s2), followed by joint 2 (1 rad/s2) and joint 4 (0.75 rad/s2). This shows that the above joints have many changes in velocity values during the movement process. Of course, these kinematics behaviors completely depend on the motion trajectory of the EE point in the workspace and the posture of robot in each specific position.

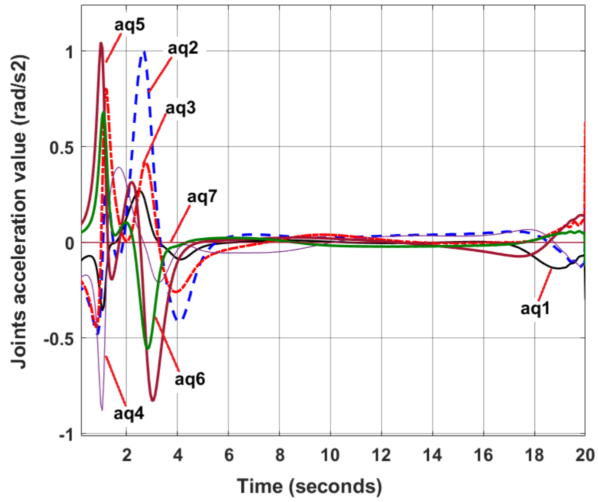

Figure 7. *Acceleration values of joints*

Fig. 8 depicts the trajectory error of the EE point in the workspace. The largest error value along the OX axis is 0.37mm, the largest error value along the OY axis is 0.32mm and the largest error value along the OZ axis is 0.26mm. Although joint 7 does not participate in movement with other joints, the trajectory error of the EE point still exists. This can be explained through the accumulated error value from previous links. It should be noted that the joints on the robot arm operate independently and the error of each joint between them certainly affects the overall error of the whole system.

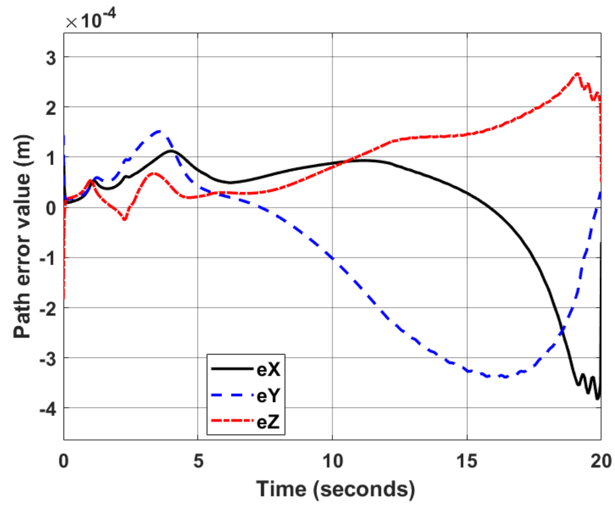

Figure 8. *Path error value of the EE point*

5. CONCLUSIONS

In general, this paper presented the analyzing results of IK problem for a robot arm with 7DOF, also known as a redundant system. In robotics, the IK problem of a redundant system is always a difficult and complex problem because it is a multi-solution problem and the choice of a reasonable solution always depends on the physical structure of the system. Through DH analysis theory, the kinematics equations are established and serves as the basis for solving the IK problem. From the input trajectory of the EE point, the joint position, velocity and acceleration values are determined using a CLF algorithm. The analysis results show that the kinematics behavior of the joints meets the requirements of their physical limits, with no singularities appearing. In particular, the path errors are very small, they meet the allowable requirements of some specific applications such as welding or 3D printing. The research results of this article have important implications in developing dynamics and control problems for redundant systems in the future.

6. REFERENCES

[1] Locke. R. C. O, Patel. R. V. *Optimal Remote Center-of-Motion Location for Robotics-Assisted Minimally-Invasive Surgery*. 2007 IEEE International Conference on Robotics and Automation, **2007**, 1990- 1905.

DOI: https:// 10.1109/ROBOT.2007.363599

[2] Lian. S, Han. Y, Wang. Y, Bao. Y, Xiao. H, Li. X, Sun. N. *Accelerating Inverse Kinematics for High-DOF Robots*. Proceedings of the 54th Annual Design Automation Conference 2017, Austin, USA (**2017**).

DOI: https:// 10.1145/3061639.3062223

[3] Yoshikawa. T. Dynamic manipulability of robot manipulators. *Journal of Robotic Systems*, **1985**, 2, 113–124.

DOI: https:// 10.1109/ROBOT.1985.1087277

[4] Wampler. C. W. *Manipulator inverse kinematic solutions based on vector formulations and damped least squares methods*. IEEE Transactions on Systems, Man, and Cybernetics, **1986**, 16, 93–101.

DOI: https://10.1109/TSMC.1986.289285

[5] Wang. L. C. T, Chen. C. C. *A combined optimization method for solving the*

[6] Sciavicco. L, Siciliano. B. A Solution Algorithm to the Inverse Kinematic Problem for Redundant Manipulators. *Journal of robotics and automation 4*, **1988**, 403-410

DOI: https://10.1109/56.804

[7] Antonelli. G, Chiaverini. S, Fusco. G. *Kinematic control of redundant manipulators with online end-effector path tracking capability under velocity and acceleration constraints*. IFAC Robot Control, Austria, **2000**, 183-188.

DOI: https://10.1109/56.804

[8] Wang. J, Li. Y, Zhao. X. Inverse kinematics and control of a 7 dof redundant manipulator based on the closed loop algorithm. *International Journal of Advanced Robotics Systems 7*, **2010**, 1 - 10.

DOI: https://10.5772/10495

[9] My. C. A, Bien. D. X, Tung. H. B, Hieu. L. C, Cong. N. V, Hieu. T. V. *Inverse kinematic control algorithm for a welding robot-positioner system to trace a 3D complex curve*. International Conference on Advanced Technologies for Communications (ATC), **2019**, 319-323.

DOI: https://10.1109/ATC.2019.8924540

[10] Aguilar. O. A, Huegel. J. C. *Inverse Kinematics Solution for Robotic Manipulators Using a CUDA-Based Parallel Genetic Algorithms*. Mexican International Conference on Artificial Intelligence **2011**, Part 1, 490-503.

DOI: https://10.1007/978-3-642-25324-9_42

[11] Husty. M. L, Pfurner. M, Schrocker. H. P. A new and efficient algorithm for the inverse kinematics of a general serial 6R manipulator. *Mechanism and Machine Theory 42*, **2007**, 66-81

DOI: https://10.1016/j.mechmachtheory.2006.02.001

[12] Kelemen. M, Virgala. I, Liptak. T, Mikova. L, Filakovsky. F, Bulej. V. A Novel Approach for an Inverse Kinematics Solution of a Redundant Manipulator. *Applied Sciences 8*, **2018**, 2-20.

DOI: https://doi.org/10.3390/app8112229

[13] Bien. D. X, Tuan. P. A, Nhat. D. D, Hiep. D. X, Nghia. T. K, Anh. M. N. Optimize the Feed Rate and Determine the Joints Torque for Industrial Welding Robot TA 1400 Based on Kinematics and Dynamics Modeling. *International Journal of Mechanical Engineering and Robotics Research*, **2020**, 9, 1335-1340.

DOI: https://10.18178/ijmerr.9.9.1335-1340

[14] F. Fahimi. *Autonomous Robots – Modeling, path planning and control*. Hardcover, Springer, **2009**.

[15] W. Jingguo, L. Yangmin and Zhao Xinhua. *Inverse Kinematics and Control of a 7-DOF Redundant manipulator Based on the Closed-Loop Algorithm*. Advanced Robotics Systems, **2010**, vol. 7, no. 4, pp. 1-10.

DOI: https://10.5772/10495